

# Link removal for control of epidemics over networks: a comparison of approaches



## Introduction

The way in which individuals and populations come into contact with each other influences disease dynamics. Information about the contact network can be used to more effectively target scarce resources to prevent disease spread.

Prior work has primarily focused on vaccinating critical nodes in the network to prevent disease spread; however, there are many diseases for which a vaccine does not exist (e.g., hepatitis C, HIV, and emerging influenza strains). For such diseases, control efforts may instead focus on modifying critical interactions between nodes.

We consider the problem of how to best prevent disease spread by removing a limited number of links in a contact network. We compare the performance of four link removal algorithms as a function of the number of links to be removed (budget) for different network structures and disease characteristics.

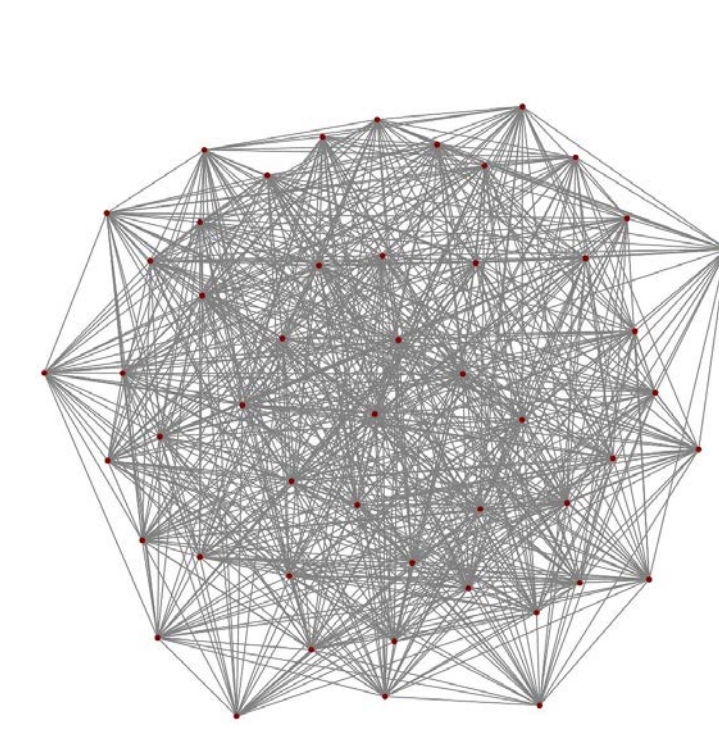
## Link removal algorithms

Characterize algorithms along two dimensions:

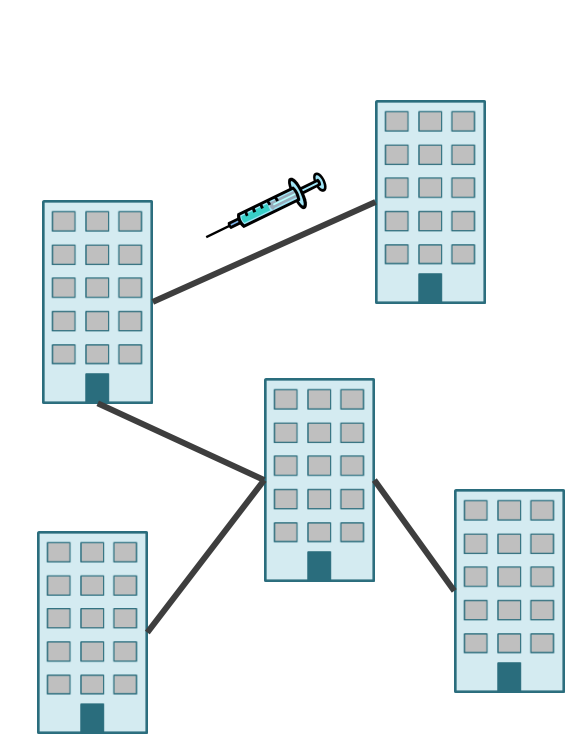
	<b>Preventive</b> Modify network prior to outbreak; No knowledge of initial infections	<b>Reactive</b> React to prevent further spread; Use knowledge of initial infections
<b>Rank-based</b> Remove links ranked according to a chosen measure of link "importance"	<b>Edge Betweenness Centrality, <math>C(e)</math></b> $C(e) = \sum_{i,j} \sigma(i,j e) / \sigma(i,j)$ where $\sigma(i,j)$ is the number of shortest paths between nodes $i$ and $j$ and $\sigma(i,j e)$ is the number of such shortest paths passing over link $e$ .	<b>Susceptible-Infected (S-I) Betweenness Edge Centrality, <math>C_{SI}(e)</math></b> $C_{SI}(e) = \sum_{i \in \mathcal{I}, j \in \mathcal{S}} \sigma(i,j e) / \sigma(i,j)$ where $\mathcal{I}$ is the set of initially infected nodes and $\mathcal{S}$ is the set of initially susceptible nodes.
<b>Optimization-based</b> Identify a set of links to remove that optimizes a chosen objective function	<b><math>R_0</math> minimization</b> <ul style="list-style-type: none"> <li>Minimize the maximum eigenvalue of the network, <math>\lambda_1(A)</math>, which is proportional to the basic reproductive number, <math>R_0</math>.</li> <li>SDP formulation, solve relaxation.</li> </ul>	<b>Optimal Quarantining</b> <ul style="list-style-type: none"> <li>Minimize the number of initially susceptible nodes "at risk" for infection (those connected to an initially infected node).</li> <li>QCQP formulation, heuristic solution.</li> </ul>

## Simulations

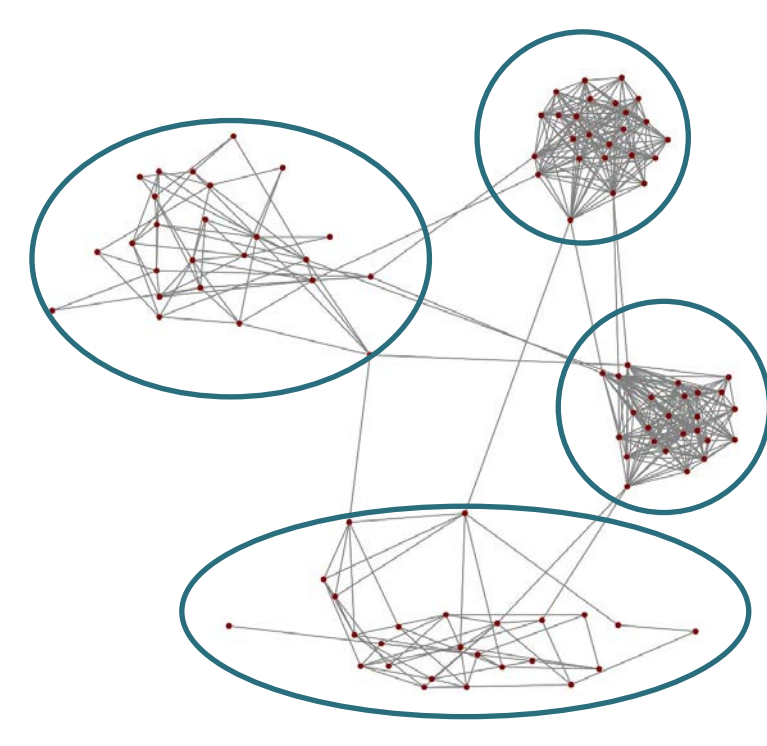
We evaluated algorithm performance for three types of networks:



**Random**  
Nodes are randomly connected with probability  $p$

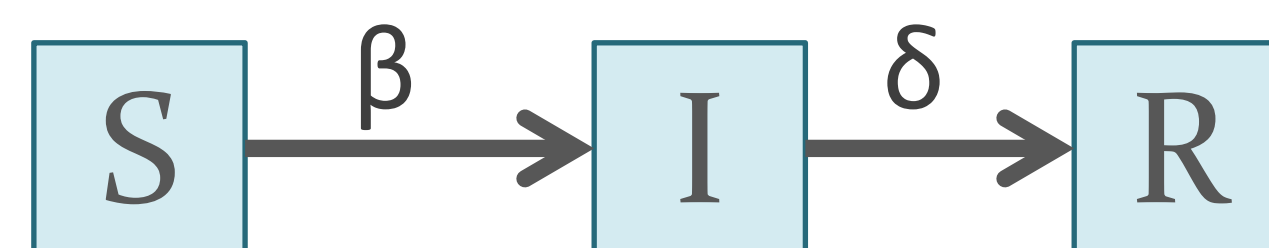


**Empirical**  
Needle-sharing between residential hotels in Winnipeg, Canada



**Community-Structured**  
Clustered communities with sparse inter-community connections

We simulated outbreaks using a Susceptible-Infected-Recovered (SIR) disease model, which is specified by  $\beta$  (transmission rate per contact) and  $\delta$  (recovery rate).



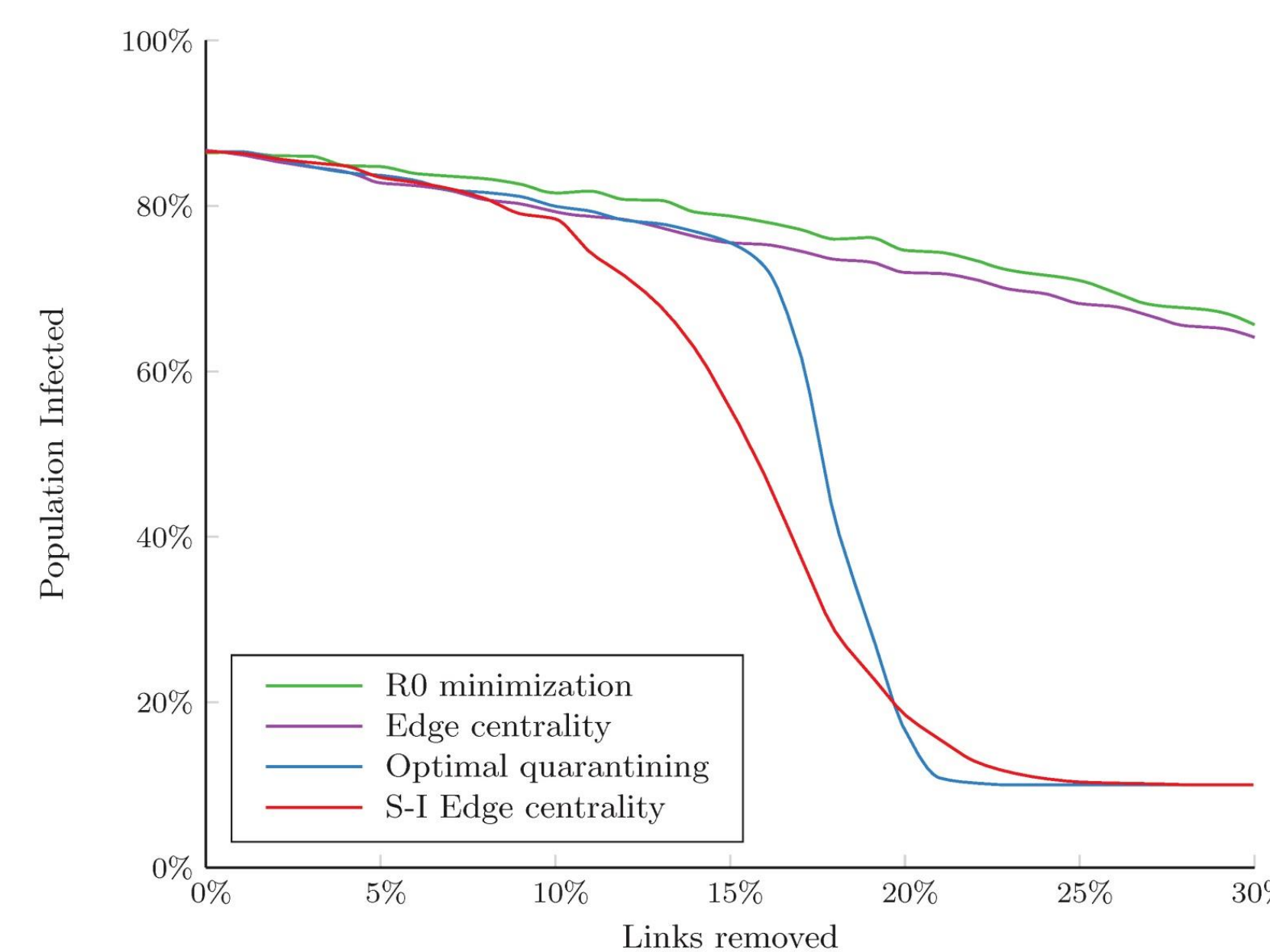
For a given network,  $\beta$  was selected such that  $R_0 = \lambda_1(A)\beta / \delta = 5$  to achieve similar outbreak severities across the different network structures.

We simulated outbreaks with 10% of nodes initially infected. We generated 300 random sets of initially infected nodes and simulated each 100 times to estimate the expected final outbreak size. Results were then averaged over the 300 sets.

## Results

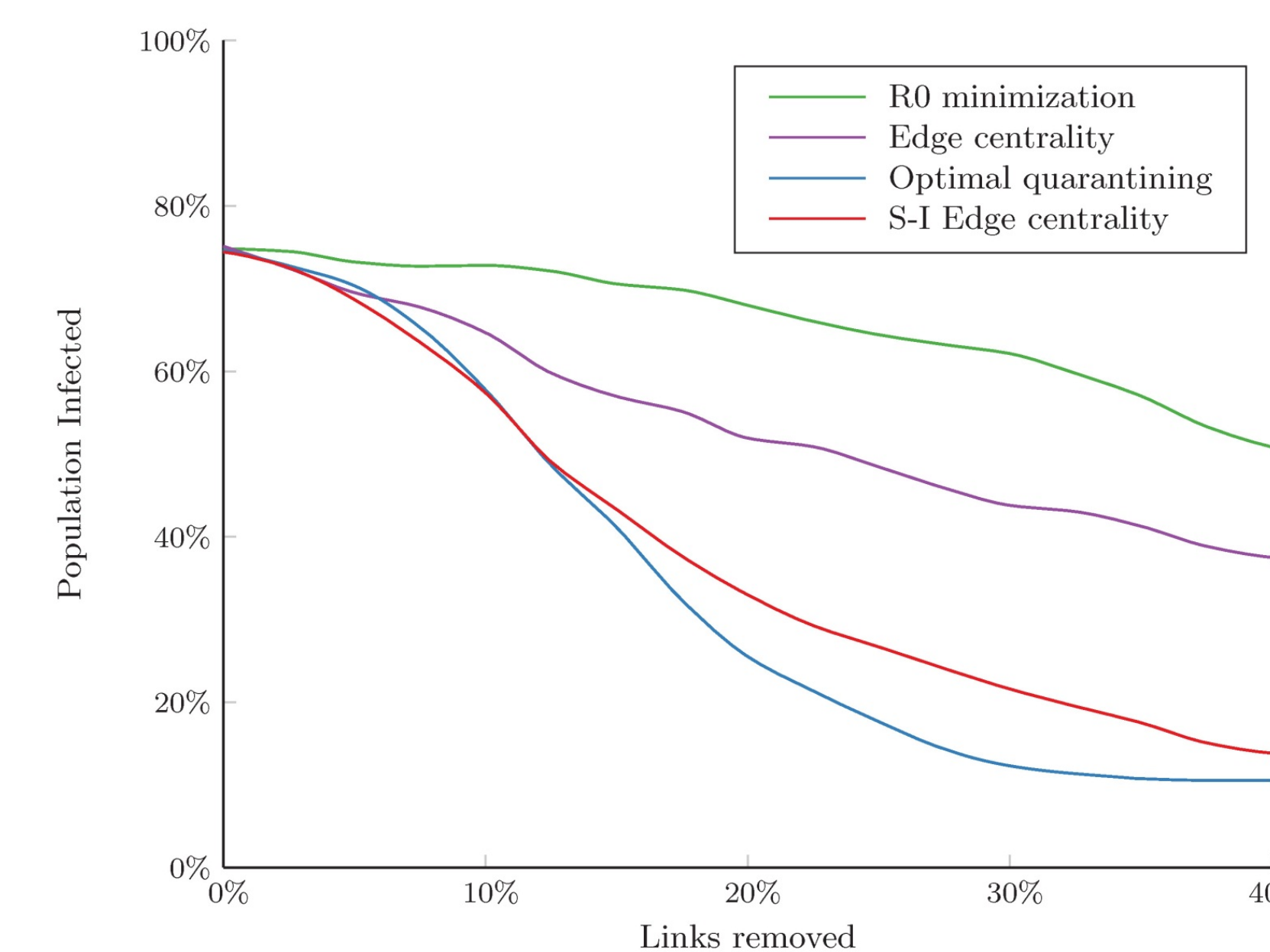
### Random

Network of 50 nodes randomly connected with probability  $p=0.40$ .



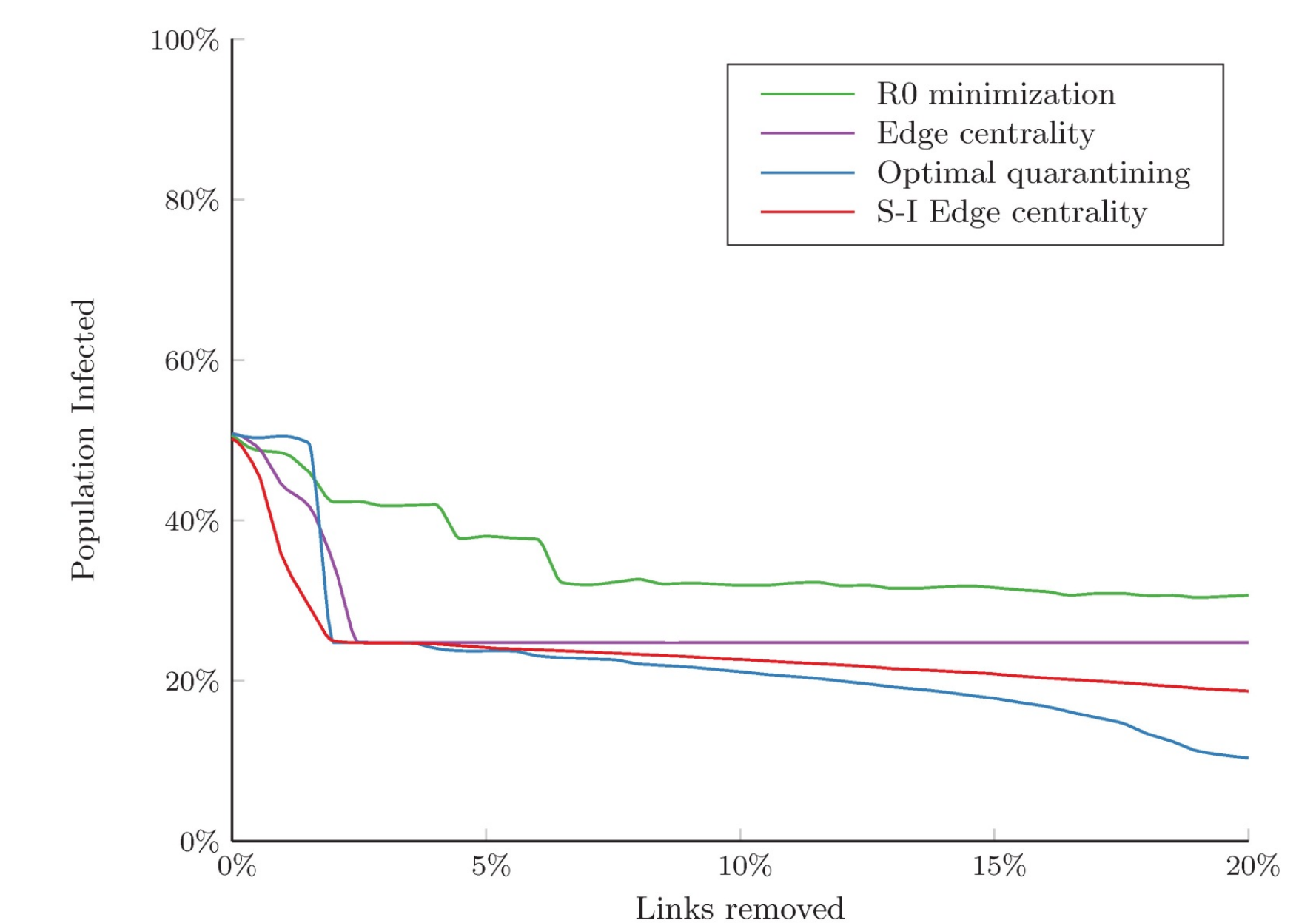
### Empirical

Network of 38 residential hotels; two hotels connected if a resident reported injecting at both.



### Community-Structured

Network of 100 nodes equally divided into 4 communities.



### Findings:

**Optimization algorithms do not always outperform greedy, rank-based approaches.**

- $R_0$  minimization averted fewer infections than removing links in order of edge centrality for all cases considered.

**Reactive algorithms consistently outperform preventive ones.**

- Knowing which nodes are initially infected is most valuable at moderate budget levels; less valuable for very small or very large budgets.

**When budgets are sufficiently large, optimal quarantining minimizes the expected outbreak size.**

- What constitutes a "sufficient" budget depends on the network structure and infectiousness of the disease.
- When the network is unstructured, disease infectiousness is low, and/or the budget is small, removing links in order of S-I edge centrality minimizes the expected outbreak size.

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